Lesson 13.2: Algebraic Techniques for finding Limits

**Recall:** Since polynomial functions are continuous everywhere, the two-sided limit exists for all values in the domain of the function. Furthermore, the two-sided limit is the same as the value of the function at that point. That is: \( \lim_{x \to c} P(x) = P(c) \) for any number \( c \).

Examples of functions whose one-sided or two-sided limits can be found by substitution:
- Identity Functions
- Linear Functions
- Quadratic Functions
- Cubic Functions
- Higher-order polynomial functions

**Note:** The limit of a constant function, \( f(x) = A \), for any value of \( c \) is \( A \).

**Algebraic Properties of Limits:**

1. Limit of a Sum: The limit of the sum of two functions is equal to the sum of their limits.

2. Limit of a Difference: The limit of the difference of two functions is equal to the difference of their limits.

**Note:** These two properties can be extended to more than two functions.
3. Limit of a Product: The limit of the product of two functions is equal to the product of their limits.

4. Limit of a Quotient: The limit of the quotients of two functions is equal to the quotient of their limits, provided that the limit of the denominator is not zero. If the denominator is zero, other strategies will be used.

5. Limit of a Power:
   a. \( \lim_{x \to c} [f(x)]^n = \left[ \lim_{x \to c} f(x) \right]^n \)
   b. \( \lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)} \)

These two properties hold true if both \( \lim_{x \to c} f(x) \) and \( \lim_{x \to c} g(x) \) are defined.

**Piecewise Functions:**

1. \( \lim_{x \to 0} f(x) = \begin{cases} x - 1, & x < 0 \\ 3x - 1, & x \geq 0 \end{cases} \)
2. \( \lim_{x \to 2^-} f(x) = \begin{cases} x^2, & x \leq 2 \\ 2x - 1, & x > 2 \end{cases} \)
3. \( \lim_{x \to 0} f(x) = \begin{cases} x^2 + 1, & x \leq 0 \\ 2x - 1, & x > 0 \end{cases} \)

Classwork: p. 963 # 22-32 even; p. 956 # 2-32 even

Homework: p. 956 # 1 – 33 odd; p. 963 # 21 – 31 odd